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Optimum Design for Minimum Mass Configuration Of Stepped Cantilever Compound Columns With Constraint On Axial Buckling Load

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Abstract--This paper presents a mathematical programming technique for obtaining minimum weight (volume) of a two stepped cantilever column, subjected to axial load constraint. Optimum configurations of circular cross-sections for isotropic column of single material and compound columns made of two different materials in each segment of step, subjected to constant axial load, are designed for the ratios of stepped diameters and segment length. This problem is optimized under equality constraints as well as specified lower and upper bounds. The evaluation of the objective function requires the solution of the buckling problem of column with variable ratios of length (L_2/L) and stepped diameters (d_1/d_2), subjected to a given axial buckling load. This problem is solved by using the Kuhn-Tucker method for the defined objective function with defined variables. Besides its accuracy yields a minor error as compared with FEM solution, overcomes the shortcomings of FEM solution, which would require resizing of elements and recomputation of their stiffness properties during optimization process.

Index Terms: **Buckling; two stepped columns; compound column; different materials; buckling load constraint; minimum mass configuration; optimum design.**

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1 INTRODUCTION

In the present study, structures made of different combinations of materials, like composite and compound structures, such as beams and plates, are taken for analysis, due to their significance in many engineering applications. Also the design of structures with minimum weight configuration has attracted the interest of researchers.

Automated design of large scale structures subjected to the frequency and buckling load constraints were dealt in the works as reported in the research works [1],[2],[3],[4]. The closed form solution in optimizing the volume of cantilever column subjected to concentrated tip load with a linear buckling load constraint is discussed in the works of J.Kiusalaas and G.B.Reddy [5] and I.Tadjbaksh and T.B.Keller [6] .

FEM based optimality criterion approach for minimum weight of a cantilever column, is reported in the works of G. Venkateswara Rao and R.Narayanaswami [7]. The present authors discussed earlier the benefits of stepped beams with two different cross sections, made of single material or two different materials through a parametric study, subjected to buckling load constraint, are reported in the research works of [8],[9]. A variation formulation is developed which forms the basis for the finite element analysis for the post buckling analysis of cantilever columns, dealt in [10]. A method is presented to obtain absolute design variables from the converged design vector obtained through the resizing formula. Lee and Choi [11] investigated thermal buckling and post buckling behavior of composite beam for the purpose of increasing the critical buckling temperature and reducing the lateral deflection for the thermal buckling. The effect of modulus ratio and length to thickness ratio on the critical thermal load was studied in the works of Aydog du [12] and Khdeir [13].

Shape optimization is done to optimize the buckling load of a Euler-Bernoulli beam, keeping volume constant, with variable stiffness, is solved by using analog equation method for the fourth -order ordinary differential equation, is presented in the works of J.T.Katsikadelis, and G.C.Tsiatas [14].

The difficulty in joining of the different materials efficiently in the manufacturing of compound columns is overcome by the modern welding techniques, namely friction welding, friction stir welding, explosive welding, cold roll welding and resistance spot welding, which are dealt in the research works reported in [17], [18], [19].

STATEMENT OF A PROBLEM

In the present study, the formulation presented is for the two stepped cantilever column of single material as shown in Fig. 1a and the compound columns of two different materials, having two segments of lengths L_1 and L_2, with free-clamped ends as shown in Fig. 1b. In the case of the two materials, the segment of length L_2 and area A_2 is made of a material having higher modulus of elasticity such as steel, copper and titanium as shown in Fig. 1b.

The admissible function for the lateral displacement 'Y' satisfying the geometric boundary conditions in non-dimensional form is given by

$$Y = a\left(1-\cos\frac{\pi x}{2L}\right) \qquad (1)$$

The boundary conditions considered for the cantilever column are

$$Y = \frac{dy}{dx} = 0 \ at \ x = 0$$

$$\frac{d^2y}{dx^2} = \frac{d^3y}{dx^3} = 0 \ at \ x = L \qquad (2)$$

By using energy method, an expression for buckling load for a cantilever compound column obtained is as

$$P = \frac{\pi^2 E_2 I_2}{4L^2}\frac{1}{\zeta+(1-\zeta)I_{12}E_{12}-\left(\frac{1}{\pi}\right)\left(\frac{1}{I_{12}E_{12}}-1\right)\sin\zeta\pi} \qquad (3)$$

Where $\zeta = L_2/L$ and $L_1+L_2=L$

The expressions for buckling load, worked out to be in non-dimensional form as

$$\lambda_{LM} = \frac{\mathrm{PL}^2}{\mathrm{E_2 I_2}} = \frac{\pi^2}{4}\frac{1}{\zeta+(1-\zeta)\frac{1}{I_{12}E_{12}}-\left(\frac{1}{\pi}\right)\left(\frac{1}{I_{12}E_{12}}-1\right)\sin\zeta\pi} \qquad (4)$$

According to the scope of this paper, the objective function for the stepped compound cantilever column as shown in Fig. 1b is

To minimize mass per unit length

$$M = \frac{\pi}{4}\rho_2\left[{d_1}^2\rho_{12}(1-\zeta)+{d_2}^2\zeta\right] \qquad (5)$$

To buckle with the maximum load,

the constraint equation for buckling axial load, $P = P^*$ can be stated as

$$P^{\Theta} = \zeta\frac{1}{{d_2}^4}+(1-\zeta)\frac{1}{E_{12}{d_1}^4}-\left(\frac{1}{\pi}\right)\left(\frac{1}{E_{12}{d_1}^4}-\frac{1}{{d_2}^4}\right)\sin\zeta\pi \qquad (6)$$

Where $P^{\Theta} = \frac{0.12118 E_2}{P^* L^2}$

The Kuhn-Tucker conditions by differentiating objective function (5) and constraint (6) with respect to the design variables d_1, d2, and ζ

We get

$$\frac{\partial M}{\partial d_1} + \lambda \frac{\partial P^{\Theta}}{\partial d_1} = 0 \quad (7)$$

$$\frac{\partial M}{\partial d_2} + \lambda \frac{\partial P^{\Theta}}{\partial d_2} = 0 \quad (8)$$

$$\frac{\partial M}{\partial \zeta} + \lambda \frac{\partial P^{\Theta}}{\partial \zeta} = 0 \quad (9)$$

Eliminating λ from equations (7) and (8) and simplifying, we get the generalized equation for the ratio of stepped diameters

$$d_{12} = \left[\frac{\zeta\left(1-\zeta-\frac{1}{\pi}\sin\zeta\pi\right)}{(1-\zeta)\left(\zeta+\frac{1}{\pi}\sin\zeta\pi\right)E_{12}}\right]^{1/6} \quad (10)$$

Solution Procedure

1. The solution of the "(10)," i.e. stepped diametral ratio, d_{12} is determined for the known values of the modulus ratio (E_{12}) by varying length ratio ($\zeta=L_2/L$).
2. The design parameter d_2 is obtained from the evaluated diametral ratio, d_{12} for the known diameter (d_1) of segment 1 of such that it is satisfying "(3)" for the known values of constrained buckling load (P^*) and area moment of inertia (I_1) of segment1.
3. Mass of the beam is calculated for the values of d_1, $d2$, ζ , ρ_2 and ρ_{12} by using "(5)".

3. As can be seen, the values in Table 2, reference values show good agreement with the present solutions. (less than 4% error)

NUMERICAL RESULTS AND DISCUSSION

Using the generalized expression for the diametric ratio (d_{12}) which expressed in the terms of length ratio ζ and modulus ratio, E_{12} developed in the preceding section , a two stepped cantilever compound column ,subjected to an end concentrated compressive load , is evaluated for the minimum weight configuration.

In the present analysis, the length of the column is taken as 0.254m (100 in) and the diameter of the aluminum segment is 30.48mm (1.2in).

The buckling load constraint (P^*) considered for all the material configurations, is 453.59 kgf (1000lb).

The material properties, namely, the Young's modulus and density of aluminum, copper, steel and titanium, considered in this analysis are presented in Table 1.

Table 2 shows comparison of the minimum weights of the two stepped cantilever compound column, made of steel or different material combinations, namely, steel-aluminum, copper-aluminum and titanium-aluminum, which give the same constraint buckling load. The minimum mass obtained for stepped steel column from the present study is 14.925kg, compared with that of through continuum analysis [6] and finite element based optimality approach [7], and found to be 3.5% and 2.4% respectively higher. The present results for minimum masses of compound columns are showing more accuracy, when compared with that of from [8], [9].

It is seen from the table 2 that for the same constraint load, the minimum mass for titanium-aluminum compound column is showing least among the other combinations of materials. And also observed to be 3.2% saving in minimum mass for stepped compound aluminum – steel column with reference to the mass of stepped steel column.

CONCLUSIONS

The optimality for minimum mass configuration developed for a two stepped cantilever column for the design variables of length ratio, diameters ratio and modulus ratio in this paper, is applicable for any combinations of two different materials. The present solutions have shown good agreement with the reference values. The advantage of this concept is that the manufacturing process involved in obtaining a desired continuous change of the area moment of inertia of the column (beam) is avoided by using compound column (beam).

The same analysis on a stepped steel column shows the advantage of using a stepped compound column. This shows that if the steps are more, the geometric constraint becomes milder and the value obtained for the optimum mass of the multi-stepped beam will be closer to the exact value, when compared to the present two stepped configuration.

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NOTATION

L_1*, *L2 : Stepped lengths of the beam

L	:	Total length of the beam
$\frac{L_2}{L}$	:	Length ratio (ζ)
d_1, d_2	:	Diameters of stepped lengths L_1 and L_2 respectively
A_1, A_2	:	Cross-sectional areas of stepped lengths L_1 and L_2 respectively
E_1, E_2	:	Young's modulii of stepped lengths L_1 and L_2 respectively
ρ_1, ρ_2	:	Densities of stepped lengths L_1 and L_2 respectively
I_1, I_2	:	Area moment of inertias of stepped lengths L_1 and L_2 respectively
d_{12}	:	Stepped diametral ratio, $\frac{d_1}{d_2}$
A_{12}	:	Area ratio, $\frac{A_1}{A_2}$
I_{12}	:	Inertia ratio, $\frac{I_1}{I_2}$
E_{12}	:	Modulus of ratio, $\frac{E_1}{E_2}$
ρ_{12}	:	Density ratio, $\frac{\rho_1}{\rho_2}$
M	:	Mass of the beam
P	:	Axial end concentrated compressive load
P^*	:	Constraint value of the buckling load
W	:	Lateral displacement
λ_{LM}	:	Non dimensional Buckling load parameter

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TABLE 1: Mechanical properties of aluminum, steel, copper and titanium.

Material	Young's Modulus GPa	Density $\frac{Kg}{m^3}$
Aluminum	**69**	**2780**
Steel	**204**	**7750**
Copper	**110**	**8940**
Titanium	***115***	***4429***

TABLE 2: Summary of minimum mass designs of two stepped cantilever columns with single /two materials subjected to buckling load

constraint of 453.5927 Kg (1000lb).

Parameter	Present study	Ref.[6]	Ref.[7]	Ref.[8]	Ref.[9]	Materials of stepped beam
Minimum mass in kg.	14.925	14.418	14.577	15.829	14.979	Steel isotropic beam
	13.970	--	--	14.005	14.183	Steel-aluminum compound beam
	20.957	--	--	21.088	21.457	copper-aluminum compound beam
	11.025	--	--	11.071	11.1768	Titanium-aluminum compound beam

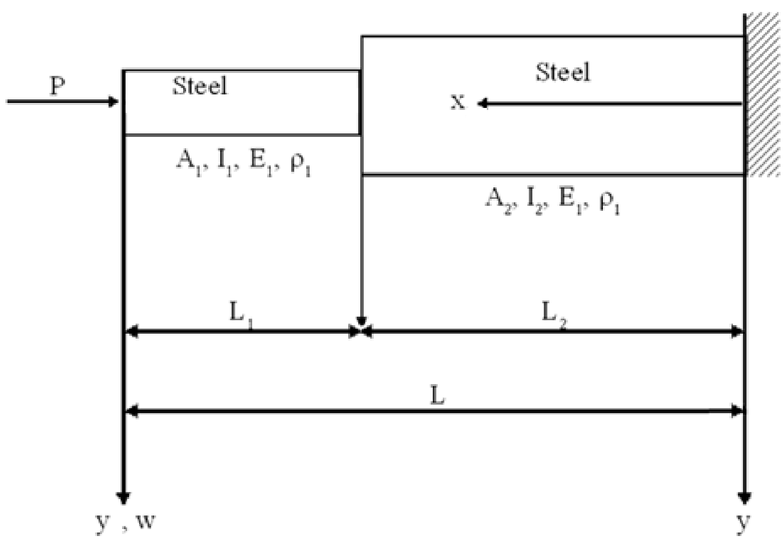

Figure 1a .Two Stepped Steel Column

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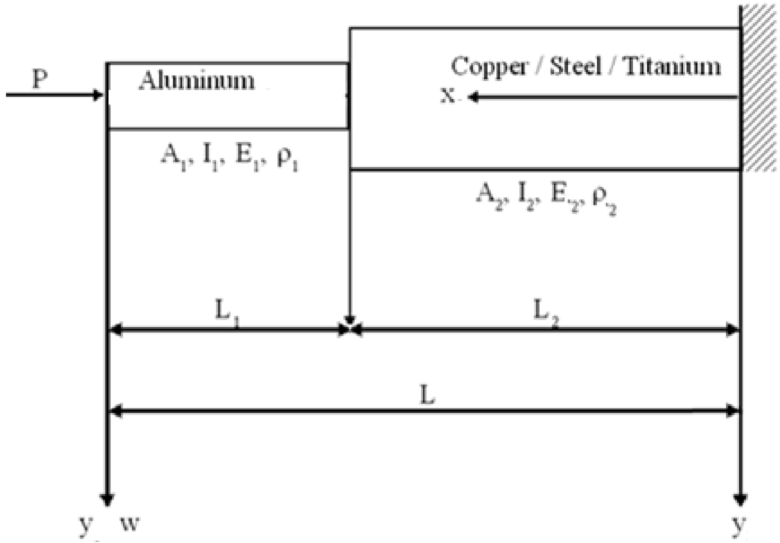

Figure 1b. Two Stepped Compound Column